Scalable k-anonymous Microaggregation: Exploiting the Tradeoff between Computational Complexity and Information Loss

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Abstract: k-anonymous microaggregation is a standard technique to improve privacy of individuals whose personal data is used in microdata databases. Unlike semantic privacy requirements like differential privacy, k-anonymity allows the unrestricted publication of data, suitable for all kinds of analysis since every individual is hidden in a cluster of size at least k. Microaggregation can preserve a high level of utility, that means small information loss caused by the aggregation procedure, compared to other anonymization techniques like generalization or suppression. Minimizing the information loss in k-anonymous microaggregation is an NP-hard clustering problem for $k \geq 3$. Even more, no efficient approximation algorithms with a nontrivial approximation ratio are known. Therefore, a bunch of heuristics have been developed to restrain high utility – all with quadratic time complexity in the size of the database at least.

We improve this situation in several respects providing a tradeoff between computational effort and utility. First, a quadratic time algorithm `ONA`* is presented that achieves significantly better utility for standard benchmarks. Next, an almost linear time algorithm is developed that gives worse, but still acceptable utility. This is achieved by a suitable adaption of the Mondrian clustering algorithm. Finally, combining both techniques a new class `MONA` of parameterized algorithms is designed that deliver competitive utility for user-specified time constraints between almost linear and quadratic.

1 INTRODUCTION

k-anonymous microaggregation is a technique designed to improve privacy of individual-related data, still keeping the data useful for research. It has been introduced by Anwar, Defays and Nanopoulus (Anwar, 1993; Defays and Nanopoulos, 1993) in 1993. Higher dimensional numerical data is clustered into groups of size at least k. We will call the result a *k-member clustering* in contrast to a *k-clustering* where the number of clusters is bounded by k. As final output each data point is represented by the centroid of its cluster and thus the modified database is k-anonymous (Samarati, 2001; Sweeney, 2002).

While k-anonymity is quite a simple condition, other more complex properties have been considered to guarantee privacy like ℓ-diversity (Machanavajjhala et al., 2007), t-closeness (Li et al., 2007) or ε-differential privacy (Dwork et al., 2006). ℓ-diversity and t-closeness may sound theoretically more appealing, but it is unclear whether and how efficiently these properties can be achieved in practice. In addition, they require a strict separation of attributes into so-called *quasi-identifiers (QI)* and *confidential attributes (CA)* which decreases the flexibility when using the anonymized data. On the other hand, differential privacy is restricted to a setting where instead of anonymizing the database and making it publicly known, only a predefined type of questions can be sent to the owner of the database. The answers protect private information by adding a suitable amount of noise depending on the diversity of the data and the type of questions. The usability is therefore limited and highly diverse data may yield quite useless answers because of larger deviations. For a more detailed discussion why these measures cannot replace k-anonymity entirely see (Li et al., 2012).

Minimizing the information loss in k-anonymous microaggregation is an NP-hard optimization problem for $k \geq 3$ (Oganian and Domingo-Ferrer, 2001; Thaeter and Reischuk, 2020). Even more, no efficient approximation algorithms with a nontrivial approximation ratio are known. Several heuristics with quadratic time complexity in the number

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of individuals have been developed to achieve k-anonymity (see e.g. (Domingo-Ferrer and Torra, 2005; Thaeter and Reischuk, 2018; Soria-Comas et al., 2019)). Quadratic time may be acceptable for small databases, but large ones with millions of individuals cannot be handled in reasonable time.

Our work tries to mitigate this problem. In 2006 LeFevre et al. introduced MONDRIAN, a clustering algorithm that achieves k-anonymity in $O(n \log n)$ time (LeFevre et al., 2006). The optimization goal of the MONDRIAN algorithm are clusters with sizes as close to k as possible. This algorithm has not been developed for microaggregation, still it creates clusterings with cluster size at least k. Hence, this strategy can be used to perform k-anonymous microaggregation by calculating and reporting centroids for each cluster created. However, the question arises whether the resulting information loss is comparable to state-of-the-art heuristics designed to minimize information loss. Our investigations have shown that this is not the case. Therefore we modify and improve this strategy to design two new algorithms MONDRIAN_V and MONDRIAN_V2D that have the same time complexity as the original one, but achieve reasonable utility. The information loss occurring is larger than that of the best quadratic time algorithms, but still on the same order. These results will be presented in section 4.

Most competitive k-anonymous microaggregation algorithms are based on the MDAV (maximum distance to average vector) principle initially formulated by Domingo-Ferrer et al. in (Domingo-Ferrer and Torra, 2005). The idea is to start with an element $\bar{x}$ of greatest distance to the centroid $c(X)$ of the whole database X and to form clusters by grouping $\bar{x}$ with its $k-1$ nearest neighbours. If less than k elements are left, the remaining elements are assigned to their closest cluster. Since the distance of many pairs of elements has to be computed this results in quadratic time complexity. While no approximation guarantees have been shown for this strategy and further improvements, these algorithms seem to perform well on benchmark databases.

In (Thaeter and Reischuk, 2018) we have proposed an extension called MDAV*. Instead of creating a new group in every step, one is given the additional option to add the current most distant element $\bar{x}$ to the closest cluster already created. The decision is made by comparing the impact on cluster cost in the local area. Without increasing the time complexity significantly the results in (Thaeter and Reischuk, 2018) show that MDAV* outperforms MDAV and other MDAV variants. Another approach named PCL has been presented in (Rebollo-Monedero et al., 2013) using clustering techniques for the case that an upper bound is given on the number of clusters – the k-means problem – instead of a lower bound on the size. However, no analysis of its computational complexity seems to have been made.

In 2019 Soria-Comas et al. presented an algorithm named ONA (Near-Optimal microaggregation Algorithm) (Soria-Comas et al., 2019). It is based on the Lloyd algorithm for efficiently clustering high dimensional data that starts with a random clustering and then iteratively improves the clustering by reassigning data points to closer clusters until a stopping condition holds (Lloyd, 1982). As the Lloyd algorithm is not tailored to guarantee a lower bound on cluster sizes it has to be modified. ONA starts with a randomly created k-member clustering and repeats the following steps for several rounds. Iterate over all elements x and consider their cluster C_x. If C_x has more than k elements try to lower the information loss by reassigning x to another cluster. If $|C_x| = k$ try to improve the clustering by dissolving C_x and redistribute its elements to other clusters nearby. Finally, split all clusters that have grown to size at least $2k$ by applying ONA recursively. This rearrangement of elements is stopped when within a round no change has occurred or a preset number of rounds has been reached.

Regarding information loss ONA seems to be comparable to previous quadratic time heuristics on real and synthetic benchmark databases. However, it was not possible for us to reproduce the excellent results claimed in (Soria-Comas et al., 2019). We have investigated how the strategy of rearranging clusters can be improved. It turned out that iterating over data points in an arbitrary order, which seems to be a good strategy for k-clustering, is not as good for k-member clustering. Instead iterating over clusters in a well chosen order is computationally more efficient and according to the benchmarks applied gives better utility. This new strategy called ONA* will be presented in section 3. Finally, by combining both methods – almost linear time complexity with a larger information loss and quadratic time with better utility – we design two new classes of algorithms called MONA$_\rho$ and MONA_2D$_\rho$ that are scalable by the parameter ρ between almost linear and quadratic time. They deliver competitive utility shown by benchmark tests in section 5.

Summarizing the results of this paper, the performance of state-of-the-art quasi-linear, resp. quadratic time heuristics for k-anonymity are significantly improved. Furthermore, we have exploited the tradeoff between computational effort and data quality providing a whole range of algorithms that suit different demands in practice.

2 PRELIMINARIES

Definition 1 (Database)**.** *A **data point, element or individual** is a d-dimensional vector $x_i = (x_i^1, \dots, x_i^d) \in \mathbb{R}^d$ of numerical attributes. A **database** $X = x_1, \dots, x_n$ is a sequence of data points, potentially including duplicates. X is k**-anonymous** if each data point occurring in X has a multiplicity of at least k.*

The common property of all microaggregation algorithms is the use of a k-member clustering to generate a partition of the data. Once clusters are defined, elements of the database are replaced by the centroid of their cluster. As a result one obtains a k-anonymous database which protects privacy of its individuals by the principle *hiding in a group of k*.

Definition 2 (k-member clustering)**.** *A k**-member clustering** of a database X is a partition $\mathcal{C}$ of its members into clusters $C_1, \dots, C_m$ such that each cluster contains at least k elements.*
*Let $\delta(x, x')$ denote the Euclidean distance between two elements and $c(C_j) = \frac{1}{|C_j|} \cdot \sum_{x \in C_j} x$ the **centroid of a cluster** C_j.*
*The **diversity of a cluster** C_j is defined as*

$$Cost(C_j) := \sum_{x \in C_j} \delta\left(x, c(C_j)\right)^2$$

*and the **total cost of a clustering** $\mathcal{C}$ by*

$$Cost(\mathcal{C}) := \sum_{C_j \in C} Cost(C_j).$$

$Cost(\mathcal{C})$ measures the closeness of elements within clusters. Once the clusters are established, creating a k-anonymous database is straight-forward by selecting the centroid as the anonymous version of each data point. Thus the data disturbance of such a procedure is related to $Cost(\mathcal{C})$ and should be as small as possible. Let us note

Fact 1: Given a clustering $\mathcal{C}$, for each cluster C_j the centroid $c(C_j)$ and $Cost(C_j)$ can be computed in $O(|C_j|\, d)$ arithmetic operations.

Hence, $Cost(\mathcal{C})$ can be estimated in time $O(n\, d)$ given $\mathcal{C}$. Note that the input size is $N = n \cdot d$ real numbers, thus this computation takes only linear time.

Definition 3 (k-anonymous microaggregation)**.** *Given a database X the k**-anonymous microaggregation problem** is to find a k-member clustering $\mathcal{C}$ with minimum cost.*

It has been shown that this is an NP-hard optimization problem for $k \geq 3$ (Oganian and Domingo-Ferrer, 2001; Thaeter and Reischuk, 2020). Even approximation algorithms with a nontrivial approximation ratio are not known. Hence, several heuristics have been developed.

To compare the data disturbance between several databases of different sizes and dimensionality, the notion *information loss* has been introduced. By dividing cost by the worst possible clustering (cluster all elements in one big cluster), one obtains a utility measure ranging from 0 for perfect utility and 1 for worst possible utility. Typically, information loss is stated as percentages, see e.g. (Soria-Comas et al., 2019).

Definition 4 (Information loss)**.** *The **diversity** $\Delta(X)$ of a database X is the sum of squared distances of all elements to the global centroid:*

$$\Delta(X) := \sum_{i=1}^{n} \delta(x_i, c(X))^2$$

*The **information loss** of a clustering $\mathcal{C}$ of X is defined as*

$$L(C, X) := \frac{Cost(\mathcal{C})}{\Delta(X)}.$$

Thus, for a given database X minimizing $Cost(\mathcal{C})$ minimizes the information loss, too.

2.1 Benchmarks and Test Setting

To compare different heuristics several benchmark databases have been used, in particular *Census*, *Tarragona* and *EIA* from the CASC project (Domingo-Ferrer and Mateo-Sanz, 2002) as well as *Cloud1*, *Cloud2*, the *Adult data set* and the *credit card clients data set* from the UCI Machine Learning Repository (Lichman, 2013). Census, Tarragona, EIA, Cloud1 and Cloud2 are relatively small databases used to compare algorithms with quadratic time complexity. The Adult and Credit Card databases are much bigger and can only be handled by subquadratic algorithms in reasonable time. More details are given in the appendix.

For a meaningful test the attributes of the databases should be standardized to mean value 0 and variance 1 prior to anonymization. This ensures that all dimensions have equal impact on the anonymization process and information loss evaluation. As microaggregation is dimension and order conserving, this standardization can be reversed after anonymization. All information losses given in this paper are expressed in percentages to be directly comparable with previously published results. The computations have been performed based on Java implementations on a PC equipped with an Intel Core i7 6850K with 4GHz core frequency and 32 GB of RAM.

3 MICROAGGREGATION IN QUADRATIC TIME WITH LOWER INFORMATION LOSS

A popular class of k-anonymous microaggregation algorithms is based on the MDAV principle explained in the introduction. The first algorithm `MDAV` has been presented in (Domingo-Ferrer and Torra, 2005). The currently best version of this methodology is `MDAV`* (Thaeter and Reischuk, 2018). Algorithm 1 gives a specification.

Algorithm 1: `MDAV`* (Thaeter and Reischuk, 2018).

```
   input : database $X$ and min cluster size $k$
   output: $k$-member clustering $\mathcal{C}$
1  Let $U \leftarrow X$; Let $\mathcal{C} \leftarrow \emptyset$
2  repeat
3      Let $\bar{x} \in U$ be the unassigned element furthest away from $c(X)$
4      Let $N_{k-1}(\bar{x}, U)$ be a cluster consisting of $\bar{x}$ and its $k-1$ nearest unassigned neighbours
5      Let $N_{k-1}(\nu(\bar{x}), U \setminus \{\bar{x}\})$ be a cluster consisting of the nearest unassigned neighbour $\nu(\bar{x})$ of $\bar{x}$ and the $k-1$ nearest unassigned neighbours of $\nu(\bar{x})$
6      Let $clos(\bar{x})$ be the closest cluster to $\bar{x}$
7      Let $Cost_1 \leftarrow \frac{Cost(N_{k-1}(\bar{x},U))}{k}$
8      Let $Cost_2 \leftarrow \frac{Cost(clos(\bar{x})+\bar{x})-Cost(clos(\bar{x}))+Cost(N_{k-1}(\nu(\bar{x}),U\setminus\{\bar{x}\}))}{k+1}$
9      if $Cost_1 \leq Cost_2$ then
10         $\mathcal{C} \cup \{N_{k-1}(\bar{x}, U)\}$
11         $U \setminus N_{k-1}(\bar{x}, U)$
12     else
13         $clos(\bar{x}) \cup \bar{x}$
14         $U \setminus \{\bar{x}\}$
15 until $|U| < k$
16 Assign each $x \in U$ to $clos(x)$
```

Our improvements build on this algorithm. Therefore let us estimate its time complexity precisely. For this we use the following facts:

Fact 2: For any data point x a list $L_{x,U}$ of the distances to all points of a set U can be computed in $O(|U|\ d)$ arithmetic operations.

Fact 3: Given $L_{x,U}$, for every $1 \leq \ell \leq |U|$ the ℓ's nearest neighbour of x and the set of its ℓ closest neighbours can be found by $O(|U|)$ comparisons.

Thus, line 3 to 5 of Algorithm 1 each take at most $O(n\ d)$ time since $|U| \leq n$. Line 6 requires $O(n\,d/k)$ steps because there can be at most n/k clusters. Computing the cost in line 7 and 8 takes time $O(k\,d)$. Thus, a single execution of the loop requires at most $O(n\ d)$ steps. The number of executions can range between n/k and n. Typically, the case generating a new cluster (line 10 to 11) should be much more likely. Thus, on average the number of executions should be on the order n/k. This gives a worst-case time bound $O(n^2 d)$ and an average bound $O(n^2 d/k)$.

The `ONA` algorithm (Soria-Comas et al., 2019) uses a different approach for k-anonymous microaggregation. Its strategy has already been presented above. A pseudo code of `ONA` is shown as Algorithm 2. While for large values of k the algorithm delivers slightly lower information loss than `MDAV` variants on benchmark databases, there are some open issues.

Algorithm 2: `ONA` (Soria-Comas et al., 2019).

```
input : database $X$ and min cluster size $k$
output: $k$-member clustering $\mathcal{C}$
Randomly generate a set of clusters $\mathcal{C} \leftarrow \{C_1, \ldots, C_m\}$ such that each cluster contains at least $k$ elements
repeat
    foreach $x \in X$ do
        Let $C_{i(x)}$ be the cluster that contains $x$
        if $|C_{i(x)}| > k$ then
            // Should $x$ be reassigned to another cluster?  (case 1)
            Extract $x$ from $C_{i(x)}$
            Compute distance between $x$ and the centroids of the clusters in $\mathcal{C}$
            Add $x$ to the cluster whose centroid is closest to $x$
        else if $|C_{i(x)}| = k$ then
            // Should cluster $C_{i(x)}$ be dissolved?  (case 2)
            For $s \in C_{i(x)}$ let $C_{j(s)}$ be the cluster with the closest centroid to $s$ among those in $\mathcal{C} \setminus C_{i(x)}$
            Let $L \leftarrow \{j(s) : s \in C_{i(x)}\}$
            Let $C'_\ell \leftarrow C_\ell \cup \{s \in C_{i(x)} : j(s) = \ell\}$, for each $\ell \in L$
            Let $Cost_1 \leftarrow Cost(C_{i(x)}) + \sum_{\ell \in L} Cost(C_\ell)$
            Let $Cost_2 \leftarrow \sum_{\ell \in L} Cost(C'_\ell)$
            if $Cost_1 > Cost_2$ then
                $\mathcal{C} \leftarrow \{C'_\ell : \ell \in L\} \cup \{C_\ell : \ell \notin (L \cup \{i(x)\})\}$
        // Split large clusters
        foreach $C \in \mathcal{C}$ do
            if $|C| \geq 2k$ then
                $\mathcal{C} \leftarrow \mathcal{C} \setminus \{C\}$; $\mathcal{C} \leftarrow \mathcal{C} \cup$ ONA$(C,k)$
until $convergence\_condition$
```

It is unclear when to stop the iteration – the conver-

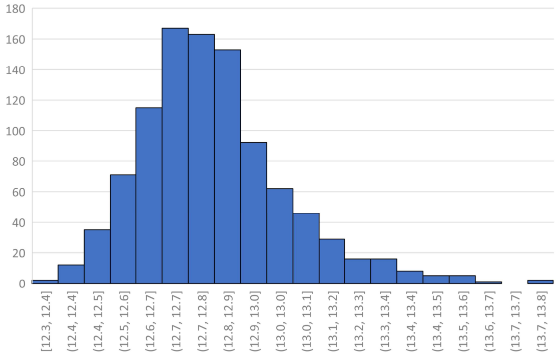


Figure 1: Histogram of the information losses of 1000 ONA runs on the Census database for $k = 10$.

gence criteria. An obvious condition is that nothing has changed within a round, but it is not clear how many rounds this may require, even more whether this situation will always be reached. In the implementation to generate the benchmark results presented below we have stopped the iteration when this condition has not been fulfilled within 30 rounds. This has happened very rarely in our tests.

Another problem is caused by the probabilistic initialization with a randomly generated k-member clustering. As for the Lloyd algorithm, a bad initialization inevitably leads to a bad output. Hence, the results may differ quite a lot and indeed, they range from better to worse than those of MDAV algorithms. In Figure 1 and Table 2 in the appendix this behavior is shown on the benchmark database *Census*.

While the authors do not provide any guidelines on how to tackle this problem, the standard approach would be to repeat the algorithm several times, let this number be μ, and output the clustering with the best solution found. As can be seen in Table 3, there is some improvement to be gained by increasing μ. But when a good confidence is aimed at, this increases the runtime significantly.

We have analyzed the methodology of generating and rearranging clusters in detail and propose a new algorithm, subsequently called ONA* that uses better selection strategies. In Table 1 ONA and ONA* are compared with a previous state-of-the-art heuristic based on the MDAV principle.

Replacing the random initial clustering by a good deterministic process increases the performance significantly,. Our experiments have shown that using an optimized variant of MDAV like MDAV* gives better results typically with lower information loss compared to the original ONA algorithm with $\mu = 100$ repetitions.

MDAV* does not guarantee a limit on the maximum cluster size, however a split is guaranteed if $2k$ or more elements are given. As a result, inputs of $3k-1$ or less elements cannot result in a cluster of size $2k$ or more. To guarantee a maximum cluster size throughout the ONA* algorithm, after initialization with MDAV* we apply the variant MDAV$^+$ (see (Thaeter and Reischuk, 2018)) to all clusters of size $2k$ or more. MDAV$^+$ delivers slightly worse information loss in general, but guarantees a maximum cluster size of $2k-1$ within the same time frame. In practice, the influence of MDAV$^+$ on ONA* is very limited, as situations in which MDAV* returns large clusters are very rare.

Concerning reassignment, ONA* makes a more precise estimation (line 22). Whereas ONA bases its decision, whether and where to move an element x, on the distances to centroids, ONA* compares the actual costs before and after a change.

A final modification simplifies matters substantially. Every cluster in step 21 of ONA has between $2k$ and $3k-1$ elements and should be divided into 2 parts. For this task ONA is not likely to find better solutions than MDAV algorithms, but requires more time and works probabilistically. Hence, we have replaced the recursive execution of ONA by a call to MDAV* which first creates 2 clusters with exactly k elements on opposite sides of the global centroid and afterwards assigns remaining elements to their closest cluster which is likely to yield an optimal solution in this special case. A complete description of ONA* is given as Algorithm 3.

To evaluate the improvements by replacing ONA with ONA*, we have performed several benchmarks on established benchmark sets (see Table 1 and Table 4). The best out of 100 ONA runs is able to outperform MDAV* in most of the tests. With bigger k, the percental difference between MDAV* and ONA becomes larger. Compared to MDAV*, ONA* is able to lower the information loss in all test cases with improvements ranging from 2% for Cloud1 and $k = 2$ to 31% for EIA and $k = 10$. The average improvement from MDAV* to ONA* is 11% over all experiments. While the average improvement from ONA to ONA* is just 3% over all experiments, its deterministic behaviour takes much lower runtime.

To determine the time complexity of ONA* consider its basic building blocks and let ζ be the number of repetitions until convergence. There are at most n/k clusters C_i of size k which might be dissolved in phase 1. For each element s of such a cluster its closest centroid $j(s)$ can be found in time $O(n/k\, d)$. For each cluster C_i evaluating the cost function for it and the at most k neighbours $C_{j(s)}$ takes time $O(k^2 d)$.

Splitting a cluster C_ℓ by MDAV* requires $O(|C_\ell|^2\, d)$ time. Each cluster C_i can give rise to at most k splits of a cluster C_ℓ of size less than $3k$, which adds up

Algorithm 3: ONA*.

```
    input  : database $X$ and min cluster size $k$
    output: $k$-member clustering $\mathcal{C}$
 1  Let $\mathcal{C} = \{C_1, \ldots, C_m\} \leftarrow$ MDAV*$(X,k)$
    // Split large clusters
 2  foreach $C \in \mathcal{C}$ do
 3      if $|C| \geq 2k$ then
 4          $\mathcal{C} \leftarrow \mathcal{C} \setminus \{C\}$; $\mathcal{C} \leftarrow \mathcal{C} \cup$ MDAV$^+(C,k)$
 5  repeat
        // Phase 1: dissolving clusters
 6      foreach $C_i \in \mathcal{C}$ with $|C_i| = k$ do
 7          For $s \in C_i$ let $C_{j(s)}$ be the cluster with the
              closest centroid to $s$ in $\mathcal{C} \setminus \{C_i\}$
 8          Let $L \leftarrow \{j(s) : s \in C_i\}$
 9          Let $C'_\ell \leftarrow C_\ell \cup \{s \in C_i : j(s) = \ell\}$, for each
              $\ell \in L$
10          Let $Cost_1 \leftarrow Cost(C_i) + \sum_{\ell \in L} Cost(C_\ell)$
11          Let $Cost_2 \leftarrow \sum_{\ell \in L} Cost(C'_\ell)$
12          if $Cost_1 > Cost_2$ then
13              $\mathcal{C} \leftarrow \{C'_\ell : \ell \in L\} \cup \{C_\ell : \ell \notin (L \cup \{i\})\}$
                // Split large clusters
14              foreach $\ell \in L$ do
15                  if $|C_\ell| \geq 2k$ then
16                      $\mathcal{C} \leftarrow \mathcal{C} \setminus \{C_\ell\}$
17                      $\mathcal{C} \leftarrow \mathcal{C} \cup$ MDAV*$(C_\ell, k)$
        // Phase 2: reassigning elements
18      foreach $C_i \in \mathcal{C}$ with $|C_i| > k$ do
19          repeat
20              foreach $s \in C_i$ do
21                  Let $C_{j(s)}$ be the cluster with the
                      closest centroid to $s$ among
                      those in $\mathcal{C} \setminus C_i$
22                  Let $improvement(s) \leftarrow$
                      $(Cost(C_i) - Cost(C_i \setminus \{s\})) -$
                      $(Cost(C_{j(s)} \cup \{s\}) - Cost(C_{j(s)}))$
23              Let $s' \leftarrow \arg\max_{s \in C_i} improvement(s)$
24              if $improvement(s') \leq 0$ then
25                  break
26              else
27                  $C_i \leftarrow C_i \setminus \{s'\}$
28                  $C_{j(s')} \leftarrow C_{j(s')} \cup \{s'\}$
                    // Split large clusters
29                  if $|C_{j(s')}| \geq 2k$ then
30                      $\mathcal{C} \leftarrow \mathcal{C} \setminus \{C_{j(s')}\}$
31                      $\mathcal{C} \leftarrow \mathcal{C} \cup$ MDAV*$(C_{j(s')}, k)$
32          until $|C_i| = k$
33  until convergence_condition
```

to $O(k^3 d)$ computational effort. Thus the total time of phase 1 can be bounded by $n/k \cdot (k \cdot O(n/k\ d) + O(k^2 d + k^3 d)) = O((n^2/k + n\,k^2)\ d)$.

For phase 2 one has to consider less than n/k clusters of size between $k+1$ and $2k-1$. The loop starting in line 19 is executed less than k times. In each execution, again for each element s of a cluster to compute its closest centroid and its improvement takes time $O(n/k\ d)$ and $O(k\ d)$ respectively. Now there can be at most one split adding time $O(k^2\ d)$. Hence, per cluster $O(n\ k\ d) + O(k^3\ d)$ time is needed. All together this gives an upper bound $n/k \cdot (O(n\ k\ d) + O(k^3\ d)) = O((n^2 + n\ k^2)\ d)$ for phase 2.

If the time $O(n^2\ d)$ for the initialization by MDAV* is added we finally get

Lemma 1. *If* ONA* *needs* ζ *iterations till convergence its runtime is bounded by* $O((n^2 + \zeta\ (n^2 + n\,k^2))\ d)$.

To establish a time bound for ONA seems to be more difficult. The time used for random initialization can be considered linear in n, a reassignment check takes time $O(n/k\,d)$ and a dissolve check $O((n+k^2)\ d)$. Iterating over n elements this already adds up to $O((n^2 + nk^2)\ d)$. This has to be multiplied by the number ζ of executions of the main loop till convergence and furthermore by the number μ of probabilistic repetitions The correct time bound may even be larger because in this calculation the recursive splitting has been ignored for which an analysis does not seem to be obvious.

4 MICROAGGREGATION IN ALMOST LINEAR TIME

In 2006 LeFevre et al. introduced MONDRIAN, an anonymization algorithm which achieves k-anonymity in $O(n \log n)$ time (LeFevre et al., 2006). The optimization goal of MONDRIAN is to create clusters with cluster sizes as close to k as possible.

Algorithm 4: MONDRIAN (LeFevre et al., 2006).

```
input  : database $X$ and min cluster size $k$
output: $k$-member clustering $\mathcal{C}$
if $|X| < 2k$ then
    return $X$
Let $dim \leftarrow$
    $\arg\max_{j \in \{1,\ldots,d\}} \left( \max_{x_i \in X} x_i^j - \min_{x_i \in X} x_i^j \right)$
Let $median \leftarrow median(\{x_i^{dim} \mid x_i \in X\})$
Let $lhs \leftarrow \emptyset$; Let $rhs \leftarrow \emptyset$
foreach $x_i \in X$ do
    if $x_i^{dim} \leq median$ then
        $lhs \leftarrow lhs \cup \{x_i\}$
    else
        $rhs \leftarrow rhs \cup \{x_i\}$
return MONDRIAN$(lhs, k) \cup$ MONDRIAN$(rhs, k)$
```

Table 1: Comparison of quadratic time microaggregation algorithms on the benchmark databases for different values of k. For ONA the best result out of 100 runs is stated, ONA*, MDAV and MDAV* are run only once as they are deterministic.

	Information Loss in % on Census					
	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 7$	$k = 10$
MDAV	3.18	5.69	7.49	9.09	11.60	14.16
MDAV*	3.17	5.78	7.44	8.81	11.37	14.01
ONA	3.44	5.47	6.92	8.16	10.08	12.45
ONA*	3.06	5.26	6.81	7.99	10.07	12.46
	Information Loss in % on Tarragona					
	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 7$	$k = 10$
MDAV	9.33	16.93	19.55	22.46	27.52	33.19
MDAV*	9.44	16.14	19.19	22.25	28.40	34.75
ONA	9.19	15.01	17.66	20.88	26.50	30.95
ONA*	9.06	15.11	17.79	20.69	26.34	31.15
	Information Loss in % on EIA					
	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 7$	$k = 10$
MDAV	0.31	0.48	0.67	1.67	2.17	3.84
MDAV*	0.22	0.45	0.62	0.91	2.03	2.63
ONA	0.21	0.40	0.59	0.81	1.60	2.01
ONA*	0.20	0.37	0.52	0.79	1.63	1.99
	Information Loss in % on Cloud1					
	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 7$	$k = 10$
MDAV	1.21	2.22	3.74	4.31	5.70	7.05
MDAV*	1.16	2.11	3.65	4.09	5.54	6.70
ONA	1.21	2.16	3.18	3.82	4.97	6.35
ONA*	1.15	2.02	3.25	3.92	5.07	6.28
	Information Loss in % on Cloud2					
	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 7$	$k = 10$
MDAV	0.68	1.21	1.70	2.03	2.69	3.40
MDAV*	0.64	1.10	1.52	1.87	2.51	3.28
ONA	0.67	1.08	1.46	1.73	2.28	2.96
ONA*	0.60	1.04	1.40	1.70	2.22	2.92

MONDRIAN is not defined as a microaggregation algorithm, but it creates a k-member clustering in the process of anonymization. Hence, this strategy can be used to perform k-anonymous microaggregation by calculating and reporting centroids for each cluster created. However, the question arises, whether the resulting information loss is comparable to state-of-the-art heuristics designed to minimize information loss. We have developed two extensions of MONDRIAN and compared the resulting runtimes and information losses to those of the original MONDRIAN algorithm as well as to MDAV* and ONA*.

The lower time complexity of MONDRIAN is caused by the fact that no distances between elements are computed. Instead, MONDRIAN resembles the process of sub-dividing a d-dimensional space by d-dimensional trees. A database is interpreted as a d-dimensional space with the elements being points in that space. In the first step, MONDRIAN splits the database into two clusters by projecting it onto one of its d dimensions and dividing elements at the median. Subsequently clusters are divided further, potentially using different *splitting dimensions* for different (sub)clusters. A cluster is no longer split and considered final, if a split at the median would result in at least one new cluster having less than k elements. Thus, in the final clustering the size of each cluster is between k and $2k-1$.

Choosing a good splitting dimension for each cluster is a crucial part of the algorithm. Especially for higher-dimensional data, choosing a less optimal splitting dimension might result in big and sparsely populated clusters, resulting in high information loss. MONDRIAN chooses the splitting dimension for any cluster as the attribute dimension with widest range of values in that cluster, a strategy aimed at reducing the area of clusters as far as possible. A pseudo code of MONDRIAN is given as Algorithm 4.

As can be seen in Table 6 in the appendix, MONDRIAN is not able to deliver information loss as low as ONA or MDAV variants. However, its computation takes far less time. Its strategy can be interpreted as acting in rounds of cutting every existing cluster of size at least $2k$ into two smaller clusters. There are $O(\log n)$ cutting rounds where every element is assigned to a new, smaller cluster. Computation of the splitting dimension is linear in d and n and computation of the median is linear in n. Hence, the total time complexity of MONDRIAN is $O(n\, d \log n)$.

Choosing splitting dimensions according to the *widest range* rule might be problematic as information loss is defined by cluster density rather than cluster area. We have investigated several alternative splitting criteria with the same asymptotic time complexity and come to the conclusion that a significant improvement could be achieved by choosing the splitting dimension as the dimension with the largest variance of values. Our resulting algorithm called MONDRIAN_V achieves 20% lower information loss on average over MONDRIAN in the test cases provided in Table 6. The splitting rule of MONDRIAN_V has the same time complexity of $O(nd)$ and can be formalized as

$$dim = \arg\max_{j\in\{1,\dots,d\}} \left(\sum_{x_i \in X} \left(x_i^j - c(X)^j \right)^2 \right).$$

The improvement going from MONDRIAN to MONDRIAN_V shows that even for low-dimensional data, the choice of the right way to cut is quite important. A natural next step is to increase the number of options for splits. Up to this point we have considered cuts according to attribute values in a single dimension only. The largest possible set of cuts would be the set of all hyperplanes dividing the database in two parts with a varying amount of elements on each side. However, deciding which splitting hyperplane to choose is a time consuming process, eliminating the performance gains made by MONDRIAN_V over ONA*.

In MONDRIAN_V splits can be interpreted as hyperplanes perpendicular to one of the unit vectors $e_1, \dots, e_d$ of the data space $\mathbb{R}^d$ dividing the elements into two clusters. The second algorithm called MONDRIAN_V2D considers additional splits. We now also allow hyperplanes that are perpendicular to a combination $e_{j_1 j_2}$ of a pair of unit vectors $e^+_{j_1 j_2} = \frac{1}{\sqrt{2}} \cdot (e_{j_1} + e_{j_2})$ and $e^-_{j_1 j_2} = \frac{1}{\sqrt{2}} \cdot (e_{j_1} - e_{j_2})$. In other words, we expand the set of possible splits by hyperplanes which are 45° and 315° between any two unit vectors. As before, splits are made at the median of the dimension (or combination of dimensions) with largest variance. Note that, by the prefactor $\frac{1}{\sqrt{2}}$ we ensure measuring variances in an orthonormal basis resulting in values comparable to those measured along original dimensions.

The number of possible splits for any given cluster increases from d to $2 \cdot \binom{d}{2} + d = d^2$ since there are $\binom{d}{2}$ pairs of dimensions to choose from and two orientations for each pair together with the d options to cut along a single dimension as before. The time complexity of MONDRIAN_V2D increases to $O(nd^2 \log n)$, but information loss further decreases by 6% on average on the Adult data set (low-dimensional data) and by 25% on average on the Credit Card data set (higher-dimensional data). A pseudo code for MONDRIAN_V2D is given as Algorithm 5.

Of course, one could extend this further and take combinations of 3 or more unit vectors increasing the time bound by additional factors of d. However, the largest gain seems to be the step from 1 to 2 dimensions.

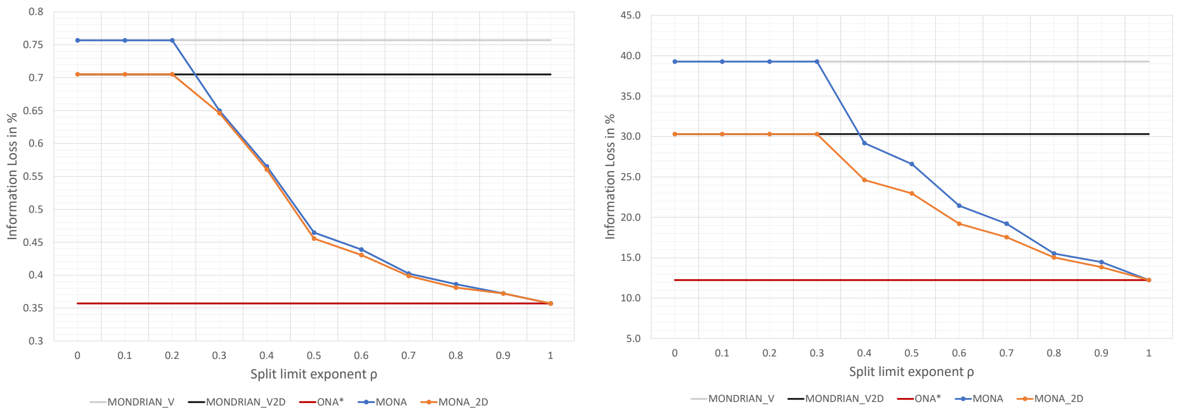
Algorithm 5: MONDRIAN_V2D.

```
input  : database $X$ and min cluster size $k$
output: $k$-member clustering $\mathcal{C}$
if $|X| < 2k$ then
    return $X$
Let
  $(dim1, dim2, o) \leftarrow \arg\max_{j_1, j_2 \in \{1,\dots,d\}, o \in \{-1,1\}}$
  $\left(\sum_{x_i \in X} \left(\frac{1}{\sqrt{2}} \cdot \left(x_i^{j_1} + o \cdot x_i^{j_2}\right)\right)\right)$
Let $median \leftarrow median(\{x_i^{dim1} + o \cdot x_i^{dim2} \mid x_i \in X\})$
Let $lhs \leftarrow \emptyset$; Let $rhs \leftarrow \emptyset$
foreach $x_i \in X$ do
    if $x_i^{dim} \leq median$ then
        $lhs \leftarrow lhs \cup \{x_i\}$
    else
        $rhs \leftarrow rhs \cup \{x_i\}$
return MONDRIAN_V2D($lhs, k$) ∪
  MONDRIAN_V2D($rhs, k$)
```

5 COMBINING ONA* AND MONDRIAN_V

As can be seen in Table 6, no MONDRIAN variant can compete with MDAV* or ONA* with respect to information loss. How can one still get the best of both worlds? We propose to combine both methods, the fast one at the beginning to split large clusters and the one of better quality for a fine grained clustering of small clusters, and name this MONA. The combination is flexible governed by a parameter ρ that can be chosen between 0 and 1. It defines the switch from MONDRIAN_V to ONA*: clusters of size larger than n^ρ are iteratively split by MONDRIAN_V, smaller ones are then handled by ONA*. Thus, we get a family of algorithms MONA$_\rho$, where MONA$_0$ equals MONDRIAN_V and MONA$_1$ is identical to ONA*. The code of MONA$_\rho$ is described in Algorithm 6. Analogously the algorithm MONA_2D$_\rho$ combines MONDRIAN_V2D and ONA*.

Since ONA* has quadratic time complexity, but is only applied to a bunch of smaller datasets, the total runtime in the ONA*-phase is reduced. Furthermore, most computation of MDAV or ONA variants is



(a) Adult database: $n = 48842$, $d = 3$, $k = 10$.

(b) Credit Card database: $n = 30000$, $d = 24$, $k = 10$.

Figure 2: Information Losses of MONA and MONA_2D for different split limits compared to MONDRIAN_V variants and ONA*on two different databases. As $n^{\rho} < 2k$ for small values of ρ, both MONA and MONA_2D behave like their MONDRIAN_V counterparts.

due to distance calculations between far apart elements. But this has little influence on the local arrangements of elements. Thus, saving these estimations in the MONDRIAN_V-phase does not increase the information loss much. Still, there might occur a decrease of data quality in the MONDRIAN_V-phase if ONA* would have clustered elements together that lie on both sides of the median of a splitting dimension used by MONDRIAN_V and now are assigned to different subproblems. However, for larger datasets such cases can be expected to have only a small influence.

In the ONA*-phase MONA$_{\rho}$ has to manage $O({}^{n}\!/\!{}_{n^{\rho}}) = O(n^{1-\rho})$ instances with input size n^{ρ} at most. The runtime of the MONDRIAN_V-phase is obviously not larger than a complete run of this algorithm. Hence, the total time complexity of MONA$_{\rho}$ can be bounded by

$$\begin{aligned} &O(nd\log n) + O\left(n^{1-\rho}\right) \cdot O((n^{2\rho} + \zeta(n^{2\rho} + n^{\rho}k^{2}))d) \\ &\quad = O(nd\log n) + O((n^{1+\rho} + \zeta(n^{1+\rho} + nk^{2}))d)\ . \end{aligned}$$

For MONA_2D$_{\rho}$ the first term gets an additional factor d. For $\rho > 0$ the first term is majorized by the second. If k is small compared to n, which for larger databases typically holds, and ζ is considered as a constant we get

Lemma 2. *For $0 < \rho \leq 1$ the runtime of* MONA$_{\rho}$ *and* MONA_2D$_{\rho}$ *is bounded by $O(n^{1+\rho}\ d)$.*

The information loss of both algorithms for different ρ are shown in Figure 2. Additionally, runtimes for MONA and MONA_2D on Credit Card are listed in Table 5.

To give a more complete overview of the performance of MONA$_{\rho}$ and MONA_2D$_{\rho}$, in Table 6 MONA$_{0.5}$ and MONA_2D$_{0.5}$ are compared to MONDRIAN_V and ONA* on Adult and Credit Card. It can be observed that MONA$_{0.5}$ and MONA_2D$_{0.5}$ deliver better results than pure MONDRIAN_V approaches. On the low-dimensional database Adult, MONA$_{0.5}$ is in reach of quadratic time algorithms like ONA* whereas MONA_2D$_{0.5}$ is not able to improve much compared to MONA$_{0.5}$. On the higher dimensional database Credit Card, MONA_2D$_{0.5}$ achieves a notable improvement to MONA$_{0.5}$. However, both have higher information loss than the quadratic time algorithms. But compared to MONDRIAN_V the improvement is significant.

Algorithm 6: MONA$_{\rho}$ (MONDRIAN_V combined with ONA*, split limit n^{ρ}).

```
input : database $X$, min cluster size $k$ and split
        limit $n^{\rho}$
output: $k$-member clustering $\mathcal{C}$
if $|X| < n^{\rho}$ then
    return ONA$^*(X,k)$
Let
  $dim \leftarrow \arg\max_{j\in\{1,\dots,d\}} \left(\sum_{x_i \in X} \left(x_i^j - c(X)^j\right)^2\right)$
Let $median \leftarrow median(\{x_i^{dim} \mid x_i \in X\})$
Let $lhs \leftarrow \emptyset$; Let $rhs \leftarrow \emptyset$
foreach $x_i \in X$ do
    if $x_i^{dim} \leq median$ then
        $lhs \leftarrow lhs \cup \{x_i\}$
    else
        $rhs \leftarrow rhs \cup \{x_i\}$
return MONA$(lhs,k,n^{\rho}) \cup$ MONA$(rhs,k,n^{\rho})$
```

6 CONCLUSION

The contribution of this paper is threefold. The ONA-approach has been optimized and transformed into a deterministic variant that works considerably faster and is at least as good as ONA w.r.t. information loss, according to the evaluation by standard benchmarks.

Further, the clustering algorithm MONDRIAN has been adapted to perform well in microaggregation applications. Two variants, namely MONDRIAN_V and MONDRIAN_V2D have been presented, both delivering superior information loss compared to MONDRIAN and operating at different ratios of performance to data quality. For lower-dimensional data the MONDRIAN technique can achieve almost the same data quality as much more time consuming algorithms.

Combining both advantages, the data quality of ONA* and the performance of MONDRIAN_V, we have designed new classes of algorithms $\mathtt{MONA}_\rho$ and $\mathtt{MONA\_2D}_\rho$ that achieve high quality data anonymization even for huge databases where quadratic time would be far too expensive.

What could be the next steps in further improving microaggregation techniques? An obvious question is whether there are even better splitting rules for MONDRIAN? On the other hand, is it possible to decrease the information loss further by spending more than quadratic time? By design, any improvement here could be applied to the $\mathtt{MONA}_\rho$ approach to get fast solutions with better quality. How well k-anonymous microaggregation can be approximated is still wide open. There is hope to achieve approximation guarantees for ONA* by carefully designing an initial clustering similar to the k-means++ algorithm (Arthur and Vassilvitskii, 2007). We plan to investigate this issue in more detail.

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APPENDIX

Benchmarks

The Census database contains 13 numerical attributes and 1080 elements. It was created using the Data Extraction System of the U.S. Bureau of Census in 2000. Tarragona contains 13 numerical attributes and 834 elements. It contains data of the Spanish region Tarragona from 1995. The EIA data set consists of 15 attributes and 4092 records. As in previous works (see e.g. (Domingo-Ferrer et al., 2006)) only a subset of 11 attributes precisely 1 and 6 to 15 have been used. Cloud1 and Cloud2 have been created using statistics from *AVHRR* images. These databases are commonly used for the training and evaluation of machine learning algorithms, and both contain 1024 elements with 10 attributes each. Adult consists of 48842 elements with 14 attributes. As for EIA, only a subset of numerical attributes namely *age*, *education number* and *hours per week* is used. This particular selection was suggested in (Rebollo-Monedero et al., 2013). It is used to test subquadratic algorithms on low-dimensional data. The Credit Card clients data set consists of 30000 elements in 24 numeric attributes. It contains inputs and predictive results from six data mining methods and is used to evaluate subquadratic algorithms on higher-dimensional data.

Experimental Results

Table 2: Statistics of the consistency of outputs on 1000 ONA executions on the Census benchmark database for different k. The information loss of MDAV* is included for reference. Information losses (IL) are stated in percentages.

	ONA on Census					MDAV* on Census
	lowest IL	highest IL	median	mean	variance	IL
$k = 2$	3.43	3.90	3.63	3.63	0.01	3.17
$k = 3$	5.42	6.24	5.70	5.71	0.01	5.78
$k = 4$	6.94	7.74	7.25	7.25	0.02	7.44
$k = 5$	8.12	9.10	8.47	8.48	0.02	8.81
$k = 7$	10.04	11.11	10.43	10.44	0.03	11.37
$k = 10$	12.36	13.66	12.77	12.80	0.04	14.01

Table 3: Statistics of the consistency of outputs on different numbers of ONA executions on the Census benchmark database for $k = 10$. The runtime and information loss of MDAV* is included for reference. As MDAV* is deterministic, it is run only once.

	ONA on Census for $k = 10$						MDAV* on Census	
	lowest IL	highest IL	median	mean	variance	runtime	IL	runtime
10 runs	12.54	13.00	12.74	12.76	0.02	1s	14.01	0s
100 runs	12.41	13.39	12.79	12.80	0.04	9s	14.01	0s
1000 runs	12.36	13.66	12.77	12.80	0.04	100s	14.01	0s

Table 4: Comparison of quadratic time microaggregation algorithms on the EIA benchmark database for different values of k. For ONA the total runtime for 100 runs is stated, ONA* and MDAV* are run only once as they are deterministic.

	Runtime on EIA					
	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 7$	$k = 10$
MDAV*	2s	1s	1s	0s	0s	0s
ONA	162s	255s	138s	327s	41s	36s
ONA*	2s	1s	1s	1s	0s	0s

Table 5: Runtimes of MONA$_\rho$ and MONA_2D$_\rho$ for different ρ and k on Credit Card. Compare with information losses stated in Figure 2b.

	Runtime on Credit Card					
	$k=2$	$k=3$	$k=4$	$k=5$	$k=7$	$k=10$
MONA$_{0.3}$	0s	0s	0s	0s	0s	0s
MONA$_{0.4}$	0s	0s	0s	0s	0s	0s
MONA$_{0.5}$	1s	0s	0s	0s	0s	0s
MONA$_{0.6}$	4s	2s	2s	2s	1s	1s
MONA$_{0.7}$	8s	5s	4s	4s	3s	3s
MONA$_{0.8}$	33s	24s	19s	16s	13s	11s
MONA$_{0.9}$	67s	48s	39s	37s	30s	25s
MONA$_1$	306s	226s	174s	146s	123s	101s
MONA_2D$_{0.3}$	2s	1s	1s	1s	1s	1s
MONA_2D$_{0.4}$	2s	1s	1s	1s	1s	1s
MONA_2D$_{0.5}$	2s	2s	2s	1s	1s	2s
MONA_2D$_{0.6}$	5s	3s	3s	2s	2s	2s
MONA_2D$_{0.7}$	8s	6s	5s	4s	4s	4s
MONA_2D$_{0.8}$	34s	24s	20s	18s	14s	12s
MONA_2D$_{0.9}$	70s	49s	42s	37s	31s	25s
MONA_2D$_1$	309s	216s	176s	146s	124s	98s

Table 6: Comparison of several microaggregation algorithms on benchmarks Adult with $d = 3$ and Credit Card with $d = 24$.

	Information Loss in % on Adult					
	$k=2$	$k=3$	$k=4$	$k=5$	$k=7$	$k=10$
MDAV*	0.04	0.09	0.14	0.18	0.28	0.42
ONA*	0.04	0.08	0.12	0.16	0.24	0.36
MONDRIAN	0.25	0.51	0.51	0.51	0.92	0.92
MONDRIAN_V	0.21	0.41	0.41	0.41	0.76	0.76
MONDRIAN_V2D	0.19	0.38	0.38	0.38	0.71	0.71
MONA$_{0.5}$	0.05	0.11	0.16	0.21	0.32	0.46
MONA_2D$_{0.5}$	0.05	0.10	0.16	0.21	0.30	0.46
	Information Loss in % on Credit Card					
	$k=2$	$k=3$	$k=4$	$k=5$	$k=7$	$k=10$
MDAV*	3.65	6.44	8.48	10.21	12.36	14.68
ONA*	3.50	5.86	7.53	8.64	10.23	12.24
MONDRIAN	30.33	30.33	41.47	41.47	43.75	50.71
MONDRIAN_V	24.05	24.05	32.54	32.54	34.12	39.27
MONDRIAN_V2D	15.81	15.81	21.93	21.93	23.23	27.34
MONA$_{0.5}$	7.74	12.56	15.99	18.53	22.45	26.59
MONA_2D$_{0.5}$	6.87	10.96	13.89	16.16	19.50	22.95
	Runtime on Adult					
	$k=2$	$k=3$	$k=4$	$k=5$	$k=7$	$k=10$
MDAV*	211s	141s	89s	77s	50s	40s
ONA*	538s	358s	267s	218s	163s	116s
MONDRIAN	0s	0s	0s	0s	0s	0s
MONDRIAN_V	0s	0s	0s	0s	0s	0s
MONDRIAN_V2D	0s	0s	0s	0s	0s	0s
MONA$_{0.5}$	1s	0s	0s	0s	0s	0s
MONA_2D$_{0.5}$	1s	0s	0s	0s	0s	0s
	Runtime on Credit Card					
	$k=2$	$k=3$	$k=4$	$k=5$	$k=7$	$k=10$
MDAV*	223s	158s	120s	95s	78s	61s
ONA*	292s	209s	174s	145s	123s	106s
MONDRIAN	0s	0s	0s	0s	0s	0s
MONDRIAN_V	0s	0s	0s	0s	0s	0s
MONDRIAN_V2D	1s	1s	1s	1s	1s	1s
MONA$_{0.5}$	1s	0s	0s	0s	0s	0s
MONA_2D$_{0.5}$	2s	2s	2s	1s	1s	2s